The Initiative for Developing Mathematical Thinking (IDMT) is an organization dedicated to assisting teachers, parents, and children in learning how to think mathematically. Developing Mathematical Thinking (DMT) is an approach to professional development for teachers based on current research of how children develop a deep and well-connected understanding of mathematics. We study how students develop mathematical ideas over time and help parents and teachers build learning environments to best promote this development. In the section “Understanding of Division” you will see how this approach works specific to that topic. In future newsletters, we will highlight similar ideas related to the learning of more math topics such as fractions, algebra, and early childhood mathematics.

You can find previous newsletters at: dmt.boisestate.edu

To make suggestions or to be added or removed from the newsletter distribution please e-mail: mti@boisestate.edu

Spring 2012 MTI Webinars

A webinar is seminar or workshop that is presented over the web. During the webinar, you will be able to interact with the instructor and other participants. We have hosted several webinars throughout the spring and having two webinars that will be conducted in April. The webinars have been archived and are available at: http://www.sde.idaho.gov/site/math/mtiWebinarsArchived.htm

In addition, Nichole Hall at the State Department of Education has set up a 1-credit option for the webinars. For more information go to: http://www.sde.idaho.gov/site/math/mtiWebinarCredit.htm

The webinars are designed to support teachers in implementing the ideas from the MTI course and the new Common Core State Standards for Mathematics (CCSS-M)

Archived Webinars:
- Composing and Decomposing Numbers
- Progression of Addition Models and Strategies
- Ratio and Proportional Reasoning
- Multiplication: Strategies, Models, Context
- Addition: Strategies, Models, Context
- Expressions and Equations
- Division: Strategies, Models, Context

Upcoming Webinars:

April 13th 5:00-6:00 (MST); 4:00-5:00 (PST)
Subtraction: Strategies, Models, Context (K-3rd)

April 18th 4:45-5:45 (MST); 3:45-4:45 (PST)
Mathematics Instructional Practices (K-8th)

For more MTI webinar information and to register, go to the MTI Follow Up website: www.tinyurl.com/mtifollowup
Understanding Division

When you think about how you learned the topic of division in school, what do you remember most? For many adults in the United States this question brings to mind countless hours in grades 4, 5, and 6 spent learning and practicing a method called long division. You probably remember your teacher explaining the steps to follow to perform long division and then assigning you 20 or more practice problems that you used to refine your technique. Maybe you were one of the students who learned long division easily, or maybe you remember it as a frustrating challenge.

With so much time spent focusing on long division in schools (in spite of many students struggling to remember all of the steps correctly) you may be surprised to learn that many mathematicians and math educators are not particularly fond of long division and have been advocating for several decades to have it de-emphasized in school mathematics. Their argument can be summed up as a practical choice educators must make between the time it takes to teach long division compared to the time it takes to teach several alternative division methods. They also weigh the benefits that long division provides compared to other methods’ benefits in later grades. Very few math specialists would claim we should ‘never’ teach long division. Instead they recommend we should treat it as one of several viable division methods…

So, what is the cost to student understanding when they learn only one series of steps and are not given the opportunity to understand the underlying rationale for those steps? Wouldn’t it be better for students to not only know how to perform a particular method but also why that method works? The alternative division methods and models shared in this article help in both of these areas. As we examine these methods you will see how they are not only accessible and easy to learn for students, more importantly, you will see how directly they connect to long division and how they might potentially help students learn long division more quickly and with a deeper understanding.

To understand why it is so important for students to learn various computational methods for division, let’s step away from mathematics for a moment and consider the topic of language arts. If you were a teacher and had to teach your class about contractions such as shouldn’t, they’re, and would’ve. Ask yourself whether you would start by teaching these specific contractions only (in isolation) or would you instead help students understand the expanded version of these words first (e.g. should not, they are, and would have) and then guide them to learn how these words can be compacted and modified to the contracted words. Even if you’ve never taught language arts before, you probably think it makes the most sense to use the latter approach and to begin instruction with the more understandable and famil-
Understanding Division Cont.

iar words in their expanded form and then teach the
more complicated contraction rules and spellings.
This is exactly the approach in mathematics that will
have the greatest long term benefit for students:
begin with the expanded sensible methods that con-
c-nect to what students already know and then help
them understand how these methods can be abbrevi-
ated to fit into the steps of a formal process like long
division.

If you’ve read our previous newsletter that focused
on multiplication, many of the division models and
methods we’ll examine will be familiar but will be
slightly different when we use them in division.

Arrays and Area Models: Visual Representations

A very valuable skill in mathematics is the ability to
visually represent a concept or mathematical pattern.
Think of these visuals as models or diagrams of what
we might be able to do using only mathematical
symbols and notation. The reason a visual model is
so important is that it not only helps communicate
what is happening in a sequence of mathematical
steps, but visuals also highlight different properties
of numbers and operations and help students under-
stand what they’re learning in ways that will help
them connect their current knowledge to new topics
learned in later grades (e.g. dividing polynomials in
algebra class).

One of the most important visual models in mathe-
matics is the array. An array is any rectangular ar-
rangement of rows and columns. If the array covers a
given space continuously with no gaps or overlaps,
then the more specific name is to call it an area mod-
el. Think about this division problem and imagine
you were modeling if with real objects or drawings:

There are 4 boxes in a stack. If there are 20 boxes,
how many stacks are there?

If you were physically modeling this problem, or
drawing a picture, a reasonable place to start would
be to make one stack of boxes that would look like

This creates an array (rectangular arrangement). Based on the problem we’re solving
(20 boxes in stacks of 4) this drawing is the array for
the division problem 20 ÷ 4. Interestingly, if the
problem was 5x4 (or 5 stacks with 4 boxes in each),
the same array would apply. In this way, arrays can
help students see the relationship between multipli-
cation and division and can help students become
familiar with different mathematical relationships.
For example, if we changed the problem to be 40
boxes total, we could just double our array and find
that it takes 10 stacks of 4 to make 40.
models, we know the measures of the rows and columns in the array (e.g. 5 columns of 4) but do not know the total area covered in the model. In the case of division, we know the total and one of the measures of the rows or columns (e.g. 20 total, in columns of 4) but we don’t know how many ‘copies’ of that measure we need to make the total. Here are examples of arrays for two more difficult division problems 252 ÷ 12 and 28.8 ÷ 2.4. In these examples, all of the individual square units have been removed making what is commonly called an open array. Open arrays are the free-hand drawings students begin using in math class once they are comfortable with using arrays and area models.

252 ÷ 12

Division Method #1: The Ratio Table

The ratio table (called the math table in early grades) is one of the most important mathematical tools children can learn. It allows students to flexibly record information they know about certain multiplication and division problems, and then use that information to find solutions to problems. Ratio tables are also a great way to record different parts of arrays students draw as they gradually build their arrays. The only specific rule that must be followed when using a ratio table is that each column must have the same multiplicative relationship (e.g. 1 to 12, 2 to 24, 10 to 120). In these examples, think of the top row in the table as the number of copies made and the bottom row as the total amount those copies would give you. Here are some ratio tables for 252 ÷ 12 and 28.8 ÷ 2.4. Look for combinations in the tables that could be put together to find the answer to the problems. This is an important skill when using the ratio table.

28.8 ÷ 2.4

Division Method #2: Partial Quotients

A quotient is the result of dividing. Partial quotients is essentially an expanded version of long division that allows children to ‘under-estimate’ the number of copies of the divisor they will remove from the number they’re dividing. This is where the name partial quotients comes from because you find the quotient in parts. It may take up more space than long division, but it offers students a chance to solve problems using what they know or find easy. It also improves number sense and encourages students to think about the place value relationships in the problem. It is an excellent way to help students learn long division with understanding as long division is just an abbreviated version of partial quotients. Here are some examples of how to use partial quotients to solve 252 ÷ 12 and 28.8 ÷ 2.4. Note that the parts of the quotient (copies of the divisor) are noted down the right side of the division bar.
**Book Corner**

*One Hundred Hungry Ants* by, Elinor J. Pinczes

**Purpose:** Students build arrays to investigate different ways to group quantities.

**Materials:** tiles, counters, beans, cubes, paper clips, noodles (100 of any item that can be counted and arranged)

How many ways can they organize their army of 100 to march to their destination? Have students organize their collections and record the different configurations on the board. Look for students who build rows and have them explain how they would describe their array.

As a teacher reading this story, have students in groups of 2 or 3 working with a collection of 100 and following the lead ant’s orders. Note which configurations were discovered by the class at the beginning.

After the story, discuss the changes that occurred in the ants marching army. Is there a way to organize the different groupings? Have them use a table to see the changes. Can they see the doubling and halving strategy? What other patterns do the students’ notice in the table? How does the use of a table help to organize the different models?

As an assessment or wrap up, have students use grid paper to model the different arrangements. They could be cut out or colored and labeled for students to have a poster or representation of one way to group a collection.

**Extensions:** Have students use a collection of 24, 36, 48 or 60. What are all the different ways to organize your collection? How do you know you found all of them? Can you organize your information into a table?

**Early Childhood Example (K–2)– The Doorbell Rang** by Pat Hutchins

**Purpose:** Students enact partitive division through use of manipulatives

**Materials:** plates, counters, paper, pencil

The story tells how the children must share the 12 cookies evenly with a growing group of friends. The story supports the enactment of dealing out cookies to ensure everyone has the same amount.

Students deal one cookie onto one plate at a time. Create a table to record the number of children and the number of cookies that they each get. As new friends arrive, students must determine the number of plates needed.

**Extension:** Students can retell the story using their own amount of cookies and friends. It is good to also introduce unequal amounts to see how students work with remainders.
Problems to Solve

K–3 Problems

K–3 division & multiplication problems should be informal opportunities for students to build their experiences with sharing, grouping, and multiplicative reasoning. Giving students multiplication and division problems in context allow the students to work at their own level. This also allows the teacher to differentiate by giving students choices for the numbers to insert into the problem by putting them in parenthesis at the end of the problem. Here are a few problems to try with your students:

* Austin brought a snack today to school. He brought four boxes of cupcakes. Each box has 5 cupcakes inside. How many cupcakes did Austin bring to school? (2, 3) (3, 7)

* Anton has been saving Box Tops to bring to school. He has 30 Box Tops saved and every time he saved 5 Box Tops he put them in a Ziploc bag. How many Ziploc bags did he use? (10, 5) (80, 10)

* I've put out your snack for today for your group to share. Count how many graham crackers your group has. Now figure out how many each person in your group will get if you make sure everyone gets the same amount?

* At the school store you can buy a sucker with two “Bronco Bucks.” So how many suckers can you buy if you have 6 “Bronco Bucks?”

4–8 Problems

Division in grades 4–8 moves from whole numbers, fractions and decimals in grades 4–6 and then to proportional reasoning in the middle grades. Consider the following problems:

* George's theater has a room with 17 rows of seats. The theater holds 204 people. How can the open array started below be used to help solve the problem?

```
  10
  7
```

* A farmer fills each jug with 2.3L of cider. If you buy 18.4L of cider, how many jugs are you buying?

<table>
<thead>
<tr>
<th>Jugs</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liters (L)</td>
<td>2.3</td>
</tr>
</tbody>
</table>

* If you can buy 20 pounds of dog food for $12, how much can you buy for $15?

These problems allow students to develop understanding of division while gaining exposure to thinking proportionally. It is important to talk with students about the relationships of the numbers to each other and the context of the problem. Using a variety of models such as the ratio table and area model can help to build understanding and make connections as they develop these concepts of multiplication and division.
6–12 Problems

There is a real struggle in math classrooms to make the connection between the operations of multiplication and division explicit without merely telling students they are inverses of each other. By the time students reach secondary mathematics, it is expected they understand this implicitly through their own practice manipulating numbers, but for many the connection is weak. Using a model, such as a ratio table may be a way to show the relationship between division and multiplication in a more tangible way while focusing on proportional reasoning skills.

Example Problem:

Jim’s new job is to make the donuts every night for the Breadbox Bakery. He is given the following instructions and told it is all he will ever need to know.

Flour, donut mix, yeast, and water are the only ingredients that you will need to make the dough for donuts You will need 10 pounds of flour, 10 pounds of donut mix, ¼ yeast, and 1 ½ gallons of water to make enough dough for about 200 donuts. Always keep the recipe proportional.

Jim knows that he will sometimes need to make more and sometimes less than 200 donuts, so he decides to create a recipe card for some various amounts of donuts. Please help him fill in the table below.

| Total Weight Dry Ingredients (Flour + Donut mix) | 20 lbs. |
| Gallons of Water | 1 ½ gal. |
| Yeast | ¼ lb. |
| Number of Donuts | 200 | 300 | 400 | 150 | 75 |

More Questions:

During his first week, Jim is asked to make 50 donuts for a special order. How much water, yeast, and dry ingredients will he need to make this batch?

One day Jim accidently puts an extra ½ gallon of water into the mixture. How much dry ingredients and yeast will he need to add in order to salvage the recipe?

Through using problems like this, students can see doubling and halving and the effects on rational numbers. Students may see going from 200 donuts to 400 donuts as doubling, but finding 300 donuts could involve either finding the values for 600 donuts and halving the recipe or reasoning that 300 is 1 ½ times larger than 200. Students will be pushed to see the relationships between multiplication and division through their work on problems like this and teachers can specifically target these big ideas during the debrief of the activity.

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Our Mission

The Initiative for Developing Mathematical Thinking (IDMT) is committed to enhancing student achievement in mathematics by offering professional development for elementary and secondary school teachers. It is our view that all students can succeed in mathematics and that teachers have perhaps the most significant impact on student learning. Therefore, we work to ensure all of our project teachers have a thorough knowledge of mathematical content, effective instructional practices, and progressions of student learning.

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